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ANALYSIS OF BALLISTIC LIMIT

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March 1978

USA ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
USA BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

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The critical impact velocity for the perforation of a target can be calculated with four techniques, two of which involve extrapolation of residual speed data and two of which involve a statistical approach. These techniques are described briefly and a nomenclature associated with each technique is suggested. The limit velocity for 30 groups of data was calculated and a comparison is made to determine their range. The results indicate that reasonable accuracy can be generally assumed for all techniques, but that an accurate critical impact velocity determination is dependent on the type and amount of data.

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TABLE OF CONTENTS

		Page
I.	INTRODUCTION.	5
II.	LIMIT VELOCITY, V_L, DETERMINATION	6
III.	PROTECTION BALLISTIC LIMIT, PBL	6
IV.	LAMBERT LIMIT VELOCITY, V_{LL}	8
V.	BALLISTIC LIMIT VELOCITY CALCULATIONS	8
	A. Mean Limit Velocity, V_{50S}, Calculation.	9
	B. Ballistic Limit Velocity, V_{50}, Calculation.	9
VI.	ANALYSIS OF DATA.	9
VII.	CONCLUSION.	13
	DISTRIBUTION.	15

I. INTRODUCTION

One of the problems encountered in the study of impact phenomena is the determination of a velocity below which an object will fail to perforate a barrier or some type of protective device. This determination is of prime importance in the design of protective structures in nuclear power plants, for the development of safety features in the automotive industry, for evaluating the effectiveness of military vehicle armor and in any problem area where an impact can cause damage. This velocity is commonly referred to as the critical impact velocity.

The techniques available to determine this velocity can be classed as either deterministic or probabilistic. In the former category, a limit velocity (denoted as V_L) is determined from physical principles (the conservation laws and material constitutive relations) but because of the complexity of the governing partial differential equations, simplifications are introduced which generally require empirical determination of one or two constants. In the probabilistic approach, models are built relying on a substantial base of data consisting of the object's striking velocity and either its residual velocity or a statement of either defeat or non-defeat of the barrier. The resulting critical velocity is most commonly expressed as a V_{50}, i.e., a striking velocity for which there exists a 50% probability of perforation of the barrier. In V_{50} determinations, a statistical approach is employed wherein the response is quantal and a sensitivity test can be applied to the data. The simplest approach requires determination of a mean velocity for three data points above and below the estimated critical impact velocity.

In practice, time and economic constraints limit the quantity of data obtained so that there is always less than that desired by either deterministic or probabilistic models for reliable estimation of critical impact velocities. After a brief discussion of various models currently in vogue and their advantages and limitations, this paper discusses procedures designed to extract reliable estimates of V_L or V_{50} from sparse data sets.

To associate the limit velocity with the technique used to obtain the critical impact velocity this report defines acronyms as follows:

- VL — a limit velocity obtained from a graphical determination or an empirical formulation.
- PBL — a protection ballistic limit determined from a special experimental technique.
- VLL — a limit velocity based on an analytical representation of striking and residual velocity data developed by Mr. J. P. Lambert of the BRL.

V_{50S} a mean limit velocity calculated by means of a simplified statistical approach.

V_{50} a ballistic limit velocity used when it is necessary to account for mixed response in the data.

II. LIMIT VELOCITY, V_L, DETERMINATION

The limit velocity, V_L[1], is determined by observing the relationship between the striking velocity, V_S, of the projectile and its residual velocity, V_R, for several striking velocity levels above an estimated critical velocity. Starting with a very high velocity the test is repeated until a very low residual velocity is observed. A plot is made of the residual velocities, V_R, as a function of the striking velocities, V_S. By drawing an "eyeball" curve through the data points and extrapolating this curve to $V_R = 0$, a value for V_S is found below which the projectile will fail to defeat the target. This striking velocity is defined as the limit velocity, V_L. Figure 1 is a graphical representation of such a curve. This method is straightforward and involves no statistical procedure. With a careful selection of striking velocities the V_L can be determined from very few tests. However, if mixed results occur as the striking velocity is reduced, then another method must be used to obtain the limit velocity.

III. PROTECTION BALLISTIC LIMIT, PBL

The protection ballistic limit, PBL[2], adds a protection criterion to the evaluation of the critical impact velocity. A complete penetration is obtained when a fragment or fragments of either the impacting projectile or armor are caused to be thrown from the plate with sufficient energy to perforate a sheet of .50mm (.020 in.) thick Dural placed parallel to, and 15cm (6 in.) to 30cm (12 in.) behind the test item. All other penetrations are partial penetrations. Since in actual experimental tests variation in results do occur, further evaluation of the type previously described must be employed.

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[1] C. [illegible] Grabarek, "Penetration of Armor by Steel and High Density Penetrators", Ballistic Research Laboratories Memorandum Report No. [illegible]124, October 1971. (AD #518394L)

[2] "Armor and Ammunition Testing", Ordnance Proof Manual, Vol. I, Aberdeen Proving Ground, MD, January 1969.

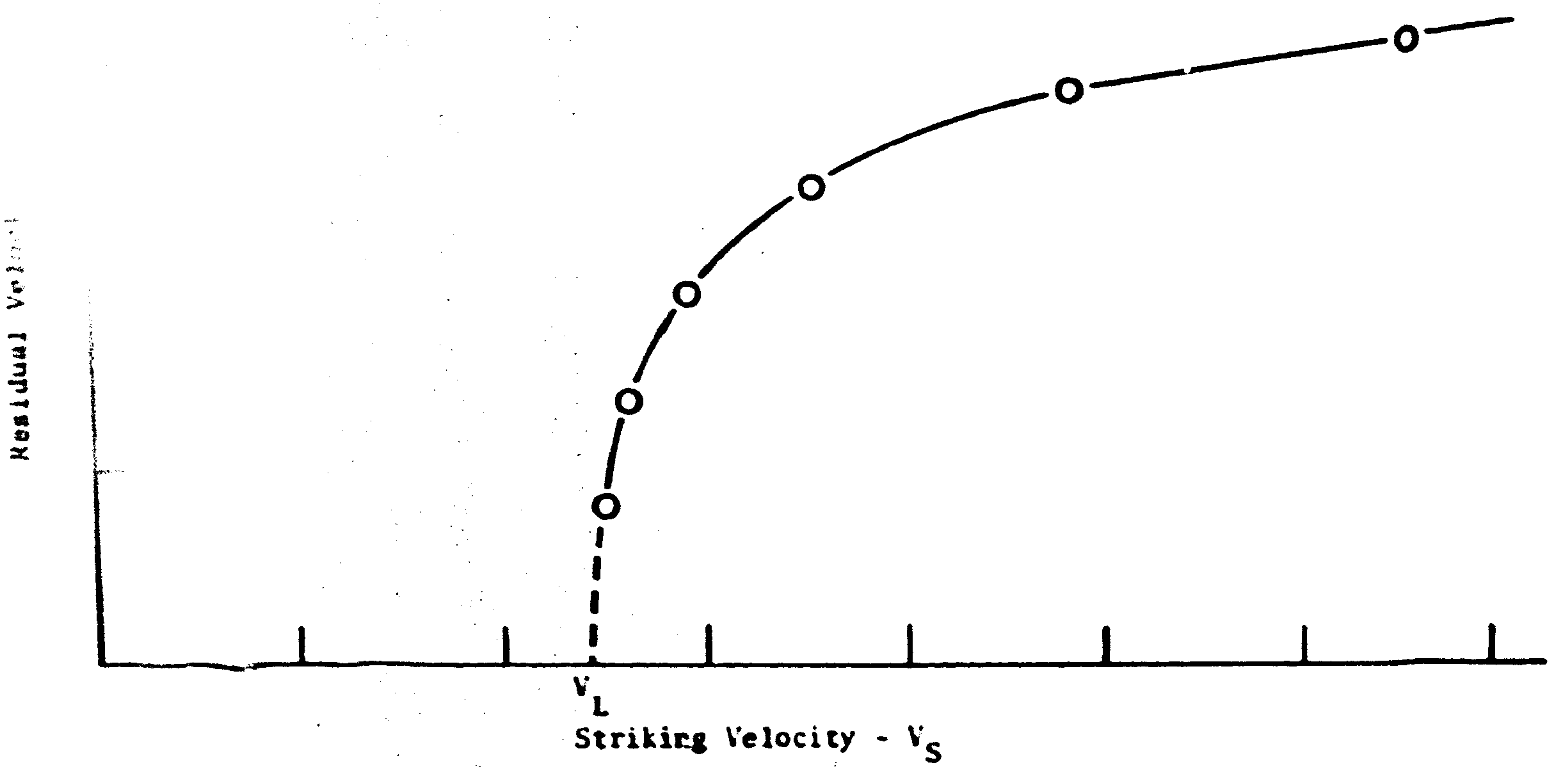

Figure 1. Graphical Determination of Limit Velocity from V_R/V_S Data.

IV. LAMBERT LIMIT VELOCITY, V_{LL}

The analytical representation of the dependence of residual velocity on striking velocity for a specified projectile-target configuration by the Lambert Method[3] is of the form:

$$V_R = \begin{cases} 0 & , \ 0 \leq V_S \leq V_{LL} \\ A \, (V_S{}^p - V_{LL}{}^p)^{1/p} & , \ V_S > V_{LL} \end{cases}$$

where the imposed constraints are:

$$p > 1 \text{ and } 0 \leq A \leq 1$$

The parameters A, p, and V_{LL} are determined by a non-linear least square regression analysis. A is the asymptote to the V_R-V_S data specified by the equation $V_R = A(V_S)$. The parameter p is the steepness factor whose value influences the curvature near the $V_R = 0$ section. The ballistic limit velocity VLL is obtained as a by-product of the fitting technique. Also, associated with the derived fit of the form to the data is the root mean square error, S, or the standard error. The advantage of this technique is that it uses the entire data whether it has or has not a mixed response.

V. BALLISTIC LIMIT VELOCITY CALCULATIONS

In experimental tests where the only observable result as a function of the striking velocity is the perforation of the target or the lack thereof, a statistical approach must be utilized to determine the ballistic limit velocity. In statistical literature this type of quantal data (yes - no, success - failure, go - no go, etc.) is denoted as "sensitivity data" where the striking velocity is the stimulus and the result is the response. In the analysis of such data a normal integrated response function is used as a model to determine the ballistic limit velocity; that is, the critical impact velocity corresponding to a fifty percent probability of success in defeating the target. In some statistical methods like the Logit[4] and Probit[5] methods the striking velocities and the number of rounds at each striking velocity are predetermined. In other methods the choices are made sequentially as the experiment progresses.

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[3] *J. P. Lambert, G. H. Jonas, "Towards Standardization in Terminal Ballistics Testing: Velocity Representation:, Ballistic Research Laboratories Report No. 1851, January 1976.*

[4] *D. J. Finney, "The Estimation of the ED 50 for a Logistic Response Curve", Sankhya, Vol. 12, Parts 1 and 2, 1952.*

[5] *D. J. Finney, "Probit Analysis-A Statistical Treatment of the Sigmoid Response Curve", Cambridge University Press, 1952.*

A. Mean Limit Velocity, V_{50S}, Calculation

A simple and straightforward approach[6] is to extract from a series of experimental results the three highest striking velocities at which no perforation occurs and the three lowest striking velocities at which perforation occurs. These data are then averaged to give a mean value of the impact velocity which is the mean limit velocity, V_{50S}. These data may be or may not be mixed, and the accuracy of the estimate will be determined by its range. If the range is small the mean limit velocity can be considered reasonable whether the response is mixed or not. As the range increases the estimate becomes unreliable.

B. Ballistic Limit Velocity, V_{50}, Calculation

In actual experimental tests the responses are generally mixed and only a rough estimate of the limit velocity can be determined by the previous method. To facilitate the analysis of sensitivity data Golub and Grubbs[7,8] of this laboratory developed a computational procedure to determine the ballistic limit velocity, V_{50}. It embodies the theory of estimation by the method of maximum likelihood developed in several statistical texts[9]. The procedure accepts sensitivity data in any form and computes the maximum likelihood estimate of the ballistic limit velocity, V_{50}, the standard deviation, the variance and covariance of the estimate, and the confidence intervals for selected probability levels.

VI. ANALYSIS OF DATA

Thirty groups of data were selected at random for calculating critical impact velocity. All the groups had V_R-V_S data, and their experimentally determined limit velocities, V_L, were included. Groups containing only quantal response were not considered because no comparisons could be made of their ballistic limits by another method.

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[6] *J. A. Feroli, "The Accuracy and Reproducibility of Several Methods for Obtaining Ballistic Limits of Armor" Development and Proof Services, Aberdeen Proving Ground, TB4-005D, AD-1200, May 1957.*

[7] *A. Golub and F. E. Grubbs, "On Estimating Ballistic Limit and Its Precision", Ballistics Research Laboratories Technical Note 151, AD 51120, March 1950.*

[8] *J. S. Hagan and V. Visnaw, "Analysis of Sensitivity Data Following A Normal Distribution", Analytical Section Report 70-A5-K3, Materiel Test Directorate, October 1970.*

[9] *W. J. Dixon and A. M. Mood, "A Method for Obtaining and Analyzing Sensitivity Data", Journal of the American Statistical Association, Vol. 43, 1948.*

The mean limit velocity V_{50S} was calculated for all the groups that had no mixed response. The V_{50S} of those groups that had less than six data points is indicated by a parenthesis and is intended only as an approximate value. The V_{50} was calculated for all the groups that had a mixed response, and the limit velocity, V_{LL}, wwas computed for all the data. The results are tabulated in Table 1.

The simplest approach yielded a mean limit velocity, V_{50S}, which approximated that of the limit velocity, V_L. Differences in the limit velocities ranged from 1 to 28 meters/second with an average difference of 7 meters/second. Since all the values of V_{50S} were included a mean limit velocity, irrespective of the number of data points used, will compare favorably with a limit velocity. The only restriction is that the data points do not show mixed results. If the response is mixed or the range of the data is too great a mean limit velocity may be calculated that is not reasonable.

The determination of the limit velocity V_{LL} by the Lambert method yields values that range from -36 to +25 meters/second with an average difference of 7 meters/second with 80% of the values being within 4 meters/second of the limit velocity V_L.

The apparent failure of the Golub-Grubbs procedure to compute a V_{50} for all tests is attributed to several restrictions imposed by the procedure. The first of these is the existence of a zone of mixed results upon which the estimate of V_{50} is dependent. A zone of mixed results exists when the highest velocity, called BIGA, at which the projectile is defeated by the target is greater than the lowest velocity, called SMALL, at which the target is defeated. When no zone exists, i.e., SMALL $\geq$ BIGA, the first estimate of the mean is zero, and other procedures such as V_{50S}, V_L, or V_{LL} must be employed. An alternate use of the procedure when no zone exists is to provide as input an initial value for the mean and sigma, and to vary both until convergence in the iteration process is obtained. However this procedure is time consuming and it is easier to use one of the other techniques. When BIGA equals SMALL its velocity is the true V_{50}. Test No. 4 demonstrates this condition. When a zone of mixed results exists the iteration process which calculates the best V_{50} estimate is dependent on sample size. If BIGA is much greater than SMALL the computation will fail. An example of such a condition would be where BIGA is the largest data point. However, a reduction in the zone of mixed results through an increase in the number of samples at higher striking velocities would insure successful computation of a V_{50}.

The second restriction is the sample size. The estimate of V_{50} is relatively independent of sample size, but the estimate of standard deviation is not only biased but extremely variable for small sample sizes and may provide questionable conclusions in the determination of confidence intervals. Results for a parametric study, using Tests No. 17 as a typical example, demonstrates the fact that increasing the sample size will provide a solution to an otherwise meaningless computation.

Table I. Critical Impact Velocity Results

Test No.	Sample Size	V_L m/s	V_{50S} m/s	V_{50} m/s	V_{LL} m/s
1	12	832		833	807
2	17	831		843	825
3	16	860		871	858
4	12	779	780		779
5	18	847		843	849
6	12	846		892	835
7	8	890		899	888
8	7	888		896	879
9	7	1234		1248	1234
10	15	732	714		720
11	17	634	(651)*		625
12	20	549	(542)		548
13	20	569	572		575
14	20	354	358		351
15	20	335	333		334
16	17	323	322		323
17	20	284		299	293
18	7	699	(709)		719
19	9	858	(867)		858
20	8	1113	(1118)		1149
21	4	1283	(1293)		1299
22	10	1452	(1455)		1450
23	15	788		810	792
24	8	933	(940)		937
25	6	1205	(1201)		1206
26	13	1417		1413	1412
27	15	1308		1318	1293
28	5	990	(993)		994
29	5	1140	(1168)		1158
30	10	1296	(1301)		1296

*( ) *means limit velocity determined with less than six data points.*

VII. CONCLUSION

The determination of the method to obtain an accurate critical impact velocity depends upon a careful examination of the data. If no zone of mixed results exists, a V_{50S}, V_L and V_{LL} can be computed irrespective of sample size. If a zone of mixed results exists, the Golub-Grubbs method will provide a reasonable value of V_{50} for the critical impact velocity for a large sample size; but the method may fail for a small sample size or for a very large zone of mixed results. Furthermore, in conducting any experiments where the results desired are in the form of sensitivity data it is advisable to use some statistical procedure to design the experiments so that the resulting data will lie within a restricted range.